Liaison Theory with a view towards Algebraic Geometry

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Abstract Liaison theory began in the XIX century as a tool to study and classify curves in $\mathbb{P}^3$ and it goes back to work of Noether, Macaulay and Severy. Roughly speaking Liaison theory is an equivalence relation among schemes of the same dimension and it involves the study of the properties shared by two schemes X_1 and X_2 whose union $X_1 \cup X_2 = X$ is either a complete intersection (CI-liaison) or an arithmetically Gorenstein scheme (G-liaison).

In the first talk, I will present classical results in CI-liaison theory which works very well for codimension 2 subschemes $X \subset \mathbb{P}^n$ and I will try to convince you that if we want to generalize them to arbitrary codimension we have to link by means of Gorenstein schemes instead of complete intersections. In other words, I will try to convince you that G-liaison theory is a much more natural approach if we want to carry out a program in arbitrary codimension.

I will describe a series of modules which are invariant under CI-liaison and I will give a number of geometric applications of these invariants. I will outline several differences between G-liaison and CI-liaison, I will prove the glicciness of standard determinantal schemes and I will end the talk with a Conjecture.

In the second lecture, I will address the minimal resolution conjecture for points on a projective variety. It is a long-standing problem in Algebraic Geometry to determine the Hilbert function of *any* set Z of distinct points on *any* projective variety $X \subset \mathbb{P}^n$. It is well-known that $H_Z(t) \leq min\{H_X(t), |Z|\}$ for any t, and that the equality holds if the points are general. A much more subtle question is to find out the exact shape of the minimal free resolution of I_Z. Mustaţă conjectured that the graded Betti numbers had to be as small as possible (when $X = \mathbb{P}^n$, we recover Lorenzini's conjecture). In my talk, I will give a brief account of the known results around Mustaţă's conjecture and address it for points on a del Pezzo surface in $\mathbb{P}^d$ and for points on a quartic surface in $\mathbb{P}^4$.

References

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